An Experimental Investigation of a Two-Slot Transmission Line on Nonplanar Surfaces

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Abstract—**The two-slot transmission line, which is the dual of a two-strip line when both lines lie in a plane, is studied experimentally when the metal sheet into which it is cut is bent through angles up to 90°. It is found that the bent two-slot line is substantially the dual of a similarly bent two-strip line including a complementary reactive junction network at the bend.**

INTRODUCTION

IN 1959 G. H. OWYANG [1] in an application of Babinet's principle experimentally and theoretically investigated the slot transmission line. The slot transmission line is the dual of the two-strip transmission line. Whereas the two-strip transmission line is ideally a pair of two-dimensional, perfectly conducting surfaces, the slot transmission line is a pair of complementary two-dimensional slots cut into a perfectly conducting, infinitely thin, infinite plane, Fig. 1(a) and (b). A summary of the results of Owyang's investigations follows:

a) The characteristic impedance of a slot transmission line ($Z_0{}^*$) is equal to the characteristic admittance (Y_0) of the complementary two-strip line.

b) The inductance per unit length L^* and the capacitance per unit length C^* of the slot system are, respectively, equal to the capacitance per unit length C and the inductance per unit length L of the two-strip line.

c) The complementary electric and magnetic fields E^* and H^* associated with the slot system are interchanged with E and H of the two-strip line.

The theoretical and experimental investigation of the planar slot transmission line is a direct application of Babinet's principle and does, in fact, meet the following restrictions on the surfaces to a close approximation [2]; the surfaces are a) infinitely thin, b) perfectly conducting, and c) infinite in extent.

In this paper the study of the slot transmission line is extended to the case when the plane containing the slot is bent, curved, or otherwise distorted out of the plane.

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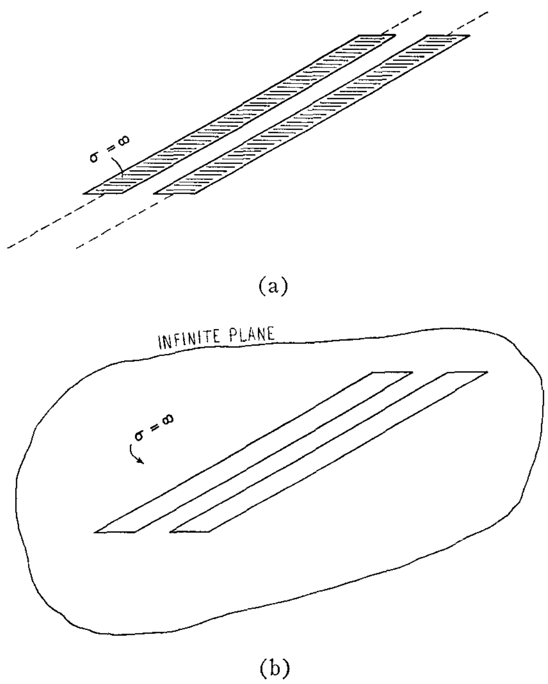
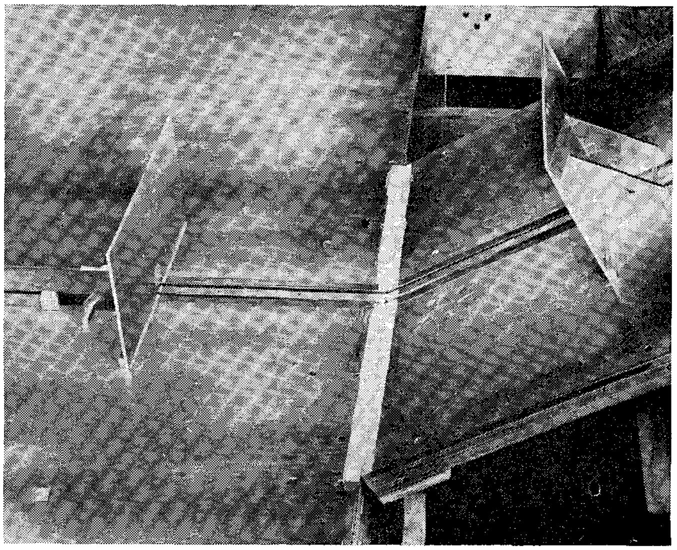


Fig. 1. (a) Two-strip line. (b) Two-slot line.

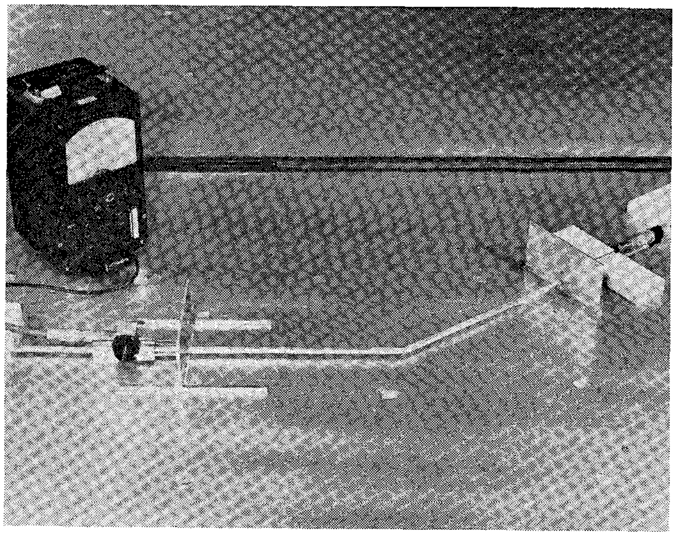
EXPERIMENTAL INVESTIGATION

Superficially it might perhaps be expected that drastic changes will occur when a slot transmission line is bent from $\theta = 0°$ (plane) to $\theta = 90°$. The first series of experiments, designed to detect changes in the standing-wave ratio and in the positions of the nulls as a function of the angle of bend using Deschamp's method, showed no discernible changes in the VSWR and only slight changes in the location of the nulls for a short-circuited line.

As a consequence, it seemed desirable to compare the behaviors of resonant sections of both a slot transmission line and a complementary two-strip transmission line [Fig. 2(a) and (b)]. If the circuits of Fig. 3 are chosen as possible equivalent transfer circuits for the bend, L_T, $L_T{}^*$, C_T, and $C_T{}^*$ may be determined much as was done by King and Tomiyasu [3] for the ordinary transmission line. L_T may, for example, be determined by positioning a short-circuiting plate a half-wavelength on each side of the bend. In this case a current maximum is maintained at the bend, emphasizing inductive and minimizing capacitive effects.

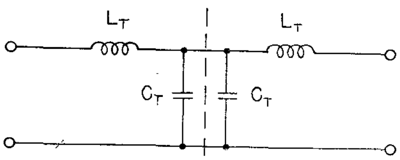


(a)

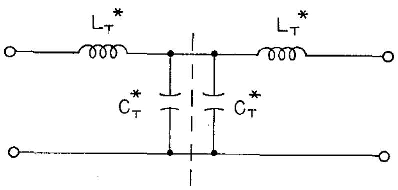


(b)

Fig. 2. (a) Resonant section of slot transmission line. (b) Resonant section of ordinary strip transmission line using mirror image.



(a)



(b)

Fig. 3. (a) An equivalent transfer circuit for ordinary transmission line at a bend. (b) An equivalent transfer circuit for slot transmission line at a bend.

In the experiment it is convenient to measure the change Δz in the distance along the line between the terminating plates when these are moved closer together or further apart to achieve a new resonance at a bend angle θ. This differential distance measured from the original position at $\theta=0$ is equal to L_T/l_0, C_T/c_0, L_T^*/l_0^*, or C_T^*/c_0^*.

As a theoretical illustration [4] consider the ordinary line with short circuits ($Z=0$) positioned at a distance $z=\lambda/2$ on each side of the bend. In this case a current maximum and charge minimum are at the bend (Fig. 4). The normalized impedance Z_{IN}/Z_0 looking either to the right or to the left is

$$Z_{IN}/Z_0 = j \tan \beta z = 0. \tag{1}$$

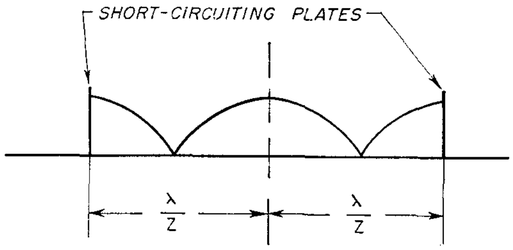
When θ (Fig. 4) is changed to θ_1, a new resonance may be obtained by moving each image plane toward the bend a distance $\Delta z/2$. The new normalized input impedance is

$$\begin{aligned} Z_{IN}/Z_0 &= j \tan \frac{2\pi}{\lambda}\left(\frac{\lambda}{2} - \frac{\Delta z}{2}\right) \\ &= -j \tan \frac{\pi \Delta z}{\lambda} \approx -j\frac{\pi\Delta z}{\lambda}. \end{aligned} \tag{2}$$

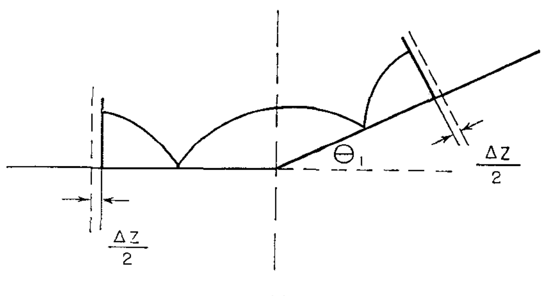
Since resonance has been achieved in this position, the bend has introduced an impedance $+j(\pi\Delta z/\lambda)$. In this case, the equivalent transfer circuit is essentially inductive with $C_T \doteq 0$ in Fig. 3 and

$$L_T/l_0 = \frac{\Delta z}{2} \text{ cm.} \tag{3}$$

Similarly, for a slot transmission line with an open circuit at $z=\lambda/2 (Z^*=\infty)$



(a)



(b)

Fig. 4. (a) and (b) Resonant section of ordinary transmission line in bending experiment.

$$Z_{IN}^*/Z_0 = -j \cot \beta z = \infty, \tag{4}$$

when $\theta = 0$. When θ is changed to θ_1, a new resonance is obtained by moving each image plane toward the bend through a distance $\Delta z/2$. The new normalized input impedance Z_{IN}^*/Z_0 is now

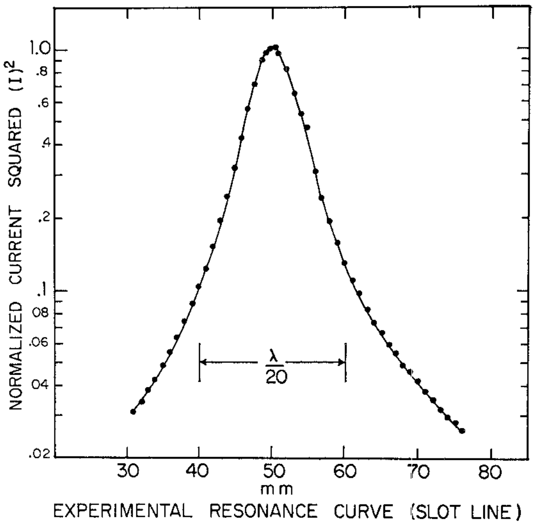


Fig. 5. Experimental resonance curve (slot line).

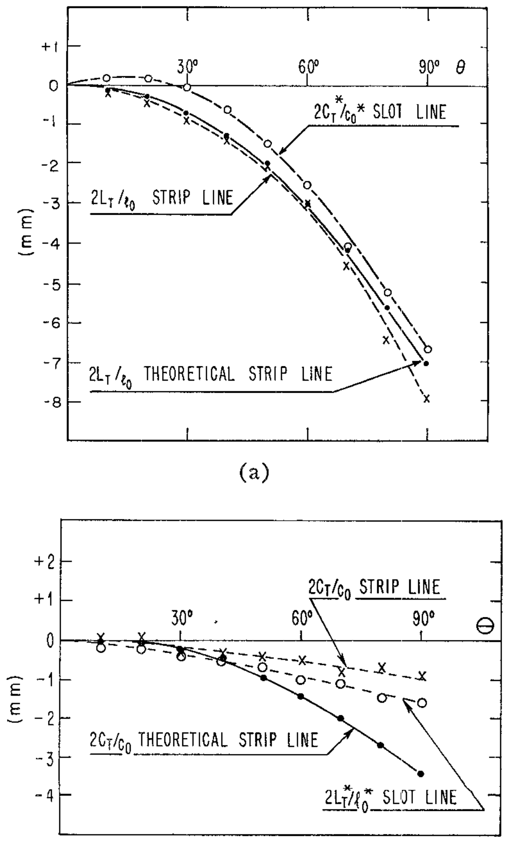


Fig. 6. (a) Graph of $2L_T l_0$ theoretical and experimental results for strip line, $2C_T^*/C_0^*$ experimental result for slot line as a function of θ, angle of bend. (b) Graph of $2C_T/C_0$ theoretical and experimental results for strip line, $2L_T^*/l_0^*$ experimental result for slot line as a function of θ, angle of bend.

$$Z_{IN}^*/Z_0 = -j \cot \left\{ \frac{2\pi}{\lambda}\left(\frac{\lambda}{2} - \frac{\Delta z}{2}\right)\right\} = \frac{1}{j \tan \frac{\pi \Delta z}{\lambda}}$$

$$\approx \frac{1}{j \frac{\pi \Delta z}{\lambda}} \cdot \tag{5}$$

Since the circuit is resonant, it is clear that the impedance $1/j(\pi\Delta z/\lambda)$ was introduced by the bend. Since the circuit is primarily capacitive, $L_T^* \doteq 0$ in Fig. 3 and

$$C_T^*/c_0^* = \frac{\Delta z}{2} \text{ cm.} \tag{6}$$

In Fig. 5 a typical experimental resonance curve for the slot line is displayed. In the experiment $\theta = 0$ and the feed plate was moved. The internal probe located near the fixed plate, detected a signal proportional to the square of the current along the outer edge of one slot. The significant results of the phase of the experimental work are displayed in Fig. 6(a) and (b) where theoretical and experimental values of $2L_T/l_0$ and $2C_T/c_0$ for a two-strip line are compared with the measured values of the complementary quantities $2C_T^*/c_0^*$ and $2L_T^*/l_0^*$ for a two-slot line. The theoretical curves were obtained by applying the approximate theory of Tomiyasu and King [3] to a transmission line with a square cross section. In all instances the differential distances moved to achieve resonance are corrected for the increase in mean length from 0 to $\pi a/4$ (a is the slot or strip thickness) as the transmission line was bent from 0° to 90°. It is clear that the two-slot line behaves substantially as the complement of a two-strip line even around bends that may be as sharp as 90°.

Conclusions

Babinet's principle offers a method of solving complementary or dual problems but only if all elements are in a plane. Electromagnetic fields resulting from dual sources or elements may be synthesized using symmetrical and antisymmetrical fields about a planar surface. No corresponding theoretical analogy exists for fields about nonplanar configurations, but the measurements here reported indicate that for a closely spaced line they do in practice obtain within reasonable approximations when bends up to 90° are made in the line. An explanation of this behavior is readily found if the slot line is considered as a transmission-line problem in which local effects are of primary importance. If a graph (Fig. 7) is made of the magnetic field $H_y(x)$ as a function of position measured normal to the slots, it is evident that the currents on the conducting surfaces are confined to an area about $\lambda/10$ in width on each side of the slots. As a consequence, it is reasonable to expect that the slot transmission line will behave as its dual does even around bends and curves since, with such a confined distribution of current, the slot transmission line is essen-

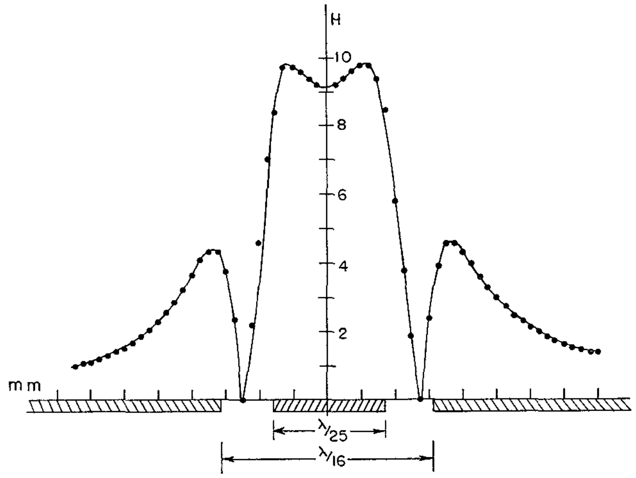


Fig. 7. Graph of normalized magnetic field H as a function of position measured close to slot line transmission.

tially a hybrid three-wire line with current directed down in the center conductor and up along the sides forming the outer edges of the slots. Alternatively, it may be considered as a four-wire line with two downward currents concentrated near the edges of the inner conductor and two return streams back along the outer conductors to the edges of the slot.

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Side-Wall-Coupled, Strip-Transmission-Line Magnetically Tunable Filters Employing Ferrimagnetic YIG Resonators

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Abstract—**This paper describes a new type of band-pass filter configuration for multiple-coupled-resonator magnetically tunable microwave filters. For two or more resonators, this configuration, which employs a coupling slot in the common side wall between two strip-transmission lines, results in the smallest possible air gap and, therefore, the least amount of leakage or fringing flux, and the smallest ampere-turns requirement on the bias magnet.**

Response curves including pass band insertion-loss, bandwidth, stop-band rejection, spurious response levels and bandwidths, VSWR, and the effect of temperature on these characteristics are presented for a two-resonator band-pass filter of the type employing YIG resonators and making use of a ferrite core electromagnet to obtain the bias field. Performance data are also given for an experimental side-wall-coupled three-resonator filter.

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Introduction

PREVIOUSLY developed types [1]–[5] of magnetically tunable filters have employed one or more single-crystal YIG (yttrium-iron-garnet) resonators in overlapping transmission-line configurations similar to those shown in Fig. 1. Filters have been built using strip-transmission line, coaxial transmission line, and waveguide. The lines or guides may be oriented at right angles as in the single resonator filter shown in Fig. 1(a), or in parallel, as in the two-resonator filter shown in Fig. 1(b).

The band-pass filters shown in Fig. 1 have in common the geometrical arrangement of resonator centers in a line along the direction of the magnetic bias field H_0. The magnet air gap length required for this arrangement of resonators increases as the number of resonators is increased. The ampere turns required to furnish this bias field increases in nearly direct proportion to this air gap. Also, the percentage of leakage flux increases as the ratio of the gap to the pole face diameter is increased.